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Innovative Design of Composite Structures: Further Studies in the Use of a Curvilinear Fiber Format to Improve Structural Efficiency

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Acknowledgements

Abstract

Further studies to determine the potential for using a curvilinear fiber format in the design of composite laminates are reported on. The curvilinear format is in contrast to the current practice of having the fibers aligned parallel to each other and in a straight line. The problem of a plate with a central circular hole is used as a candidate problem for this study. The study concludes that for inplane tensile loading the curvilinear format is superior. The limited results to date on compression buckling loads indicate that the curvilinear designs are poorer in resisting buckling. However, for the curvilinear design of interest, the reduction in buckling load is minimal and so overall there is a gain in considering the curvilinear design.

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Introduction

Conventional design philosophies for fiber-reinforced composite structures are based on the idea of using multiple layers of fibers embedded in a matrix, the fibers in each layer being straight and aligned in a particular direction. Though each layer may have its own unique fiber orientation, the idea of allowing the fiber orientation within a layer to vary from point to point has not been seriously considered. Fabrication techniques and analysis procedures have precluded this form of fiber usage, herein referred to as a curvilinear format, and have emphasized the straightline format. Part of the reason for the lack of usage has been the complication in manufacturing structures with the curvilinear format. Contemporary fiber handling techniques, however, now make this issue less of an obstacle. Another part of the reason for the lack of usage is the difficulty in analyzing and understanding the response of such structures. For the curvilinear format, any analysis relying on solving the governing differential equations would be complicated by the fact that coefficients in the equations would be functions of the spatial variables. This could be viewed as an obstacle for not considering a curvilinear format. However, an analysis involving numerical computations would be no more difficult than one commonly used to study the response of structures using the conventional straightline fiber format. With the latter fact in mind, the real issue is revealed: If the fibers do not have to remain in straight lines, how should their orientation vary from point to point? To conduct an analysis, numerical or closed-form, the fiber angles must be known. The fiber angles to use depends on the use of the structure, its geometry, and its loading. However,

these same issues govern the directions of the fibers in the various layers in the conventional straightline format. A particular problem must be considered before an answer can be obtained. A previous report [1] began to address the issue of exploiting the curvilinear fiber format. The report addressed the question of improving the response of a square plate with a central circular hole subjected to an inplane tensile load. The criterion for improvement was to increase the load capacity of the plate. This was accomplished by allowing the fiber orientation to vary from point to point within the plate. The response of the plate was calculated using a finite-element analysis and a material failure criterion, specifically the maximum strain failure criterion. A major component of the analysis, and indeed the major point of the study, was an iteration scheme that computed the fiber direction from point to point in the plate such that the fiber direction was aligned with the principal stress direction. Since the plate was discretized because of the finite-element method, the analysis reduced to finding the fiber direction in each element, the direction within an element being considered constant. Because the properties of fiber-reinforced materials are orthotropic, using the principal stress directions based on an isotropic material was not correct. Therefore, the iteration started with the isotropic principal stress directions and then used incremental changes in the fiber direction from point to point until the principal stress directions and the fiber directions did coincide at each point (actually within each element). Only one plate geometry was considered. The plate consisted of 16 layers of AS4/3501 graphite-epoxy with a square planform and hole diameter to plate width ratio, D/W, of 1/3. Several curvilinear designs were considered, iteration being used in all cases to find the fiber direction for that particular design. These curvilinear designs were compared to more conventional straightline format designs. The specific details reported on will not be reviewed at this point. Suffice it to say that the study indicated that substantial gains in tensile load capacity could be realized by using the curvilinear design rather than the conventional straight line design. As just mentioned, these predictions were based on

using the maximum strain criterion. Because the predicted increases in performance were substantial, it was decided that the analysis should be redone using another failure criterion. If the different failure criterion predicted similar increases in performance, then the arguments for a curvilinear format could be made on a somewhat firmer basis. In addition, a range of geometries needed to be considered. And finally, though the tensile capacity was predicted to improve, compression loadings are equally important and an important issue was the effect of the curvilinear design on the compression response. Since the laminates being studied were thin, the primary compression response to consider was buckling.

The present report describes the predictions using a second failure criterion to measure performance in the tensile loading case. In addition, the influence of geometry on the predicted improvement, and some preliminary predictions of the influence of the curvilinear design on the buckling loads are reported. The second failure criterion chosen is the Tsai-Wu tensor polynomial criterion. This criterion was chosen because it is an interacting stress-based criterion, whereas the maximum strain criterion is a strain-based noninteracting criterion. Results from the previous report are presented here for comparison with the results based on the Tsai-Wu failure criterion.

The next section describes the details of the problem being studied, and presents numerical results for four plate geometries and several straightline and curvilinear designs. The results are in the form of the tensile load capacity for the various designs, curvilinear and conventional, with attention being given to the improvement in capacity using the curvilinear design. The section following that discusses the preliminary calculations regarding buckling.

Problem Details

Figure 1 illustrates the planform geometry of the plate under consideration. The plate width is denoted by W, its length by L, and the hole diameter by D. The plate is loaded in the lengthwise direction by a tensile force distributed uniformly over each end. With two values of D/W and two values of L/W, four distinct plate geometries are considered. The values of D/W are 1/3 and 1/6. The values of L/W are 1 and 2. These values are felt to represent moderate extremes in the plate geometries encountered in practice. The primary reason for considering various geometries is to determine if the conclusions regarding improved performance for the curvilinear design depend strongly on plate geometry. Referring to fig. 1, the +x axis is to the right, the +y axis vertical. Fiber angles are defined relative to the +x axis.

The response of the plates is computed using a previously written finite-element code. The basic element is an eight-node isoparametric element and with nine Gauss integration points. Due to symmetry in the geometry, loading, and material properties, a one-quarter plate analysis can be used. Figures 2-5 show the meshes for the four geometries considered. Each mesh consists of 192 elements, 12 of which are around the hole edge. The material properties used in the study are chosen to closely represent AS4/3501. The elastic properties are given by:

$$E_1 = 19.9 \times 10^6 \text{ psi}; \quad E_2 = 1.28 \times 10^6; \quad G_{12} = 1.03 \times 10^6; \quad \nu_{12} = 0.298$$
$$\text{lamina thickness} = 0.005 \text{ in.} \tag{1}$$

The failure strains are taken to be:

tensile failure in fiber direction, $\varepsilon_{1T} = 10.5 \times 10^{-3}$

compression failure in fiber direction, $\varepsilon_{1C} = 10.5 \times 10^{-3}$

tensile failure perpendicular to fiber, $\varepsilon_{2T} = 5.8 \times 10^{-3}$ (2)

compression failure perpendicular to fiber, $\varepsilon_{2C} = 23.0 \times 10^{-3}$

inplane shear failure, $\gamma_S = 13.1 \times 10^{-3}$.

The failure stresses are taken to be

tensile failure in fiber direction, $\sigma_{1T} = 210$ ksi

compression failure in fiber direction, $\sigma_{1C} = 210$ ksi

tensile failure perpendicular to fiber, $\sigma_{2T} = 7500$ ksi (3)

compression failure perpendicular to fiber, $\sigma_{2C} = 29.8$ ksi

inplane shear failure, $\tau_S = 13.5$ ksi.

Numerical Results

Tables 1 - 5 present the predicted tensile failure load for a variety of plate designs. Each of the first four tables presents results for a specific plate geometry and a variety of straightline and curvilinear designs. These tables present the failure load based on both the maximum strain criterion and the Tsai-Wu criterion. The maximum strain criterion is considered the primary criterion and the Tsai-Wu criterion the backup. Therefore, the Tsai-Wu data are in parenthesis. In each of the first four tables the failure loads have been normalized by the failure load of a conventional straightline quasi-isotropic design of that particular geometry based on the maximum strain criterion. If a normalized failure load is greater than 1, then the design is better than the conventional quasi-isotropic design. If the normalized failure load is less than 1, the design is not as good as the quasi-isotropic design. With this format the influence of design on the failure load is readily evident. Each of the first four tables also presents remarks as to the location and mode of failure. The mode of failure is determined by which part of the five-part maximum strain criterion governs the load. The fifth table combines information from the first four tables and compares geometries and straightline and curvilinear designs. In that table the failure loads are normalized by the failure load of the conventional design quasi-isotropic rectangular plate (L/W = 2) with the smaller hole (D/W = 1/6). In this table the dependence, or lack thereof, of the curvilinear and conventional designs on geometry is evident.

Table 1 presents the results for plates with L/W = 2 and D/W = 1/6. In reality, the results are presented for a plate 20 in. long and 10 in. wide with a hole diameter of

1.67 in. The maximum strain failure load of a quasi-isotropic laminate with this configuration is predicted to be 27.6 Kips/in. The laminate is predicted to fail due to the fibers in the 0° layers at the net section failing. The Tsai-Wu criterion predicts a very similar load level. At this time the Tsai-Wu criterion has not been interrogated to determine its predicted failure mode.

With that as a basis, a unidirectional plate - i.e., a $(0_8)_s$ plate - is examined next. Such a plate is highly unusual except it has an obvious counterpart if curvilinear designs are considered. The counterpart is design 3, the plate denoted as $(C_8)_s$, the C denoting curvilinear. As can be seen, the unidirectional plate fails at a load 17% below the conventional quasi-isotropic design due to a shear failure. This shear failure occurs somewhat away from the net section. Such failures are often seen in fiber-reinforced composites, the failure resulting in a crack running parallel to the fibers from the net section hole edge to the end of the plate. The failure occurs due to the rapid change in direction of the stress trajectories as they transmit the load around the hole. Any time there is a gradient in one stress component (as occurs here when the uniform load on the end of the plate must become nonuniform to accommodate the hole), the other stress components must develop a gradient. If the stress equilibrium equations are interpreted as algebraic equations in stress gradients, this becomes evident. Here, though there are no shear stresses at the ends of the plate, they must develop as the hole is approached. Since a unidirectional material is very weak in shear, the design is shear limited. The Tsai-Wu criterion also predicts reduced performance for the unidirectional plate.

The $(C_8)_s$ design overcomes the weakness in shear by having the fibers aligned with the principal stress directions at every point in the plate. In the principal material system there are no shear stresses whatsoever. This design is 26% stronger than the quasi-isotropic design and considerably stronger than the unidirectional configuration. This design is also limited by matrix properties, the plate failing due to ex-

cess strain perpendicular to the fibers. This strain is due to a Poisson effect near the net section. This is a result of the following: At the net section hole edge the fibers are parallel with the load direction and the strains in the fiber are quite high. These high tensile strains cause contraction strains perpendicular to the fibers - i.e., in the y direction. Still at the net section but away from the hole edge the fibers are parallel with the load direction but the tensile strains are not as high. As a result, the contraction strains are not as high. Due to this difference in contraction strains moving out the net section away from the hole edge, the material fails perpendicular to the fibers. This clearly is not utilizing fiber-reinforcing to its utmost advantage.

Design 4 is the obvious answer to preventing failure of the material perpendicular to the fibers in the curvilinear design. This design is designated a $(O/C_7)_s$ laminate. In this case the O is an 'oh' not a 'zero' and it stands for orthogonal. It means that two of the 16 curvilinear layers in the $(C_8)_s$ design are allowed to become everywhere orthogonal with the load-bearing curvilinear layers to prevent the curvilinear layers from separating due to the above-mentioned tensile strain perpendicular to the fibers. As seen by the normalized failure load, such a design is very effective. The design is predicted to react twice as much load as the quasi-isotropic design. The failure is due to the fibers failing at the net section hole edge. Such a design exploits the full strength potential of the fibers. The Tsai-Wu criterion predicts a similar increase in strength relative to the quasi-isotropic design.

Unfortunately, the $(O/C_7)_s$ design is impractical. First, it may be very difficult to fabricate a laminate that has the required orthogonal grid of fibers. Fabricating several layers with the same curvilinear format seems reasonable. However, having to fabricate other layers that are everywhere orthogonal to the first group of layers may be unreasonable. Second, and more important, if an attempt is made to fabricate an orthogonal grid of fibers, there is no doubt the results would be less than perfect alignment. This lack of perfect alignment would introduce shear which would then

lead to failure. Third, and equally as important, if there is any shear whatsoever transmitted to the plate, the plate would fail. The orthogonal design is ideal for pure axial load but any deviation from this loading would lead to large shear stresses which the design could not tolerate.

The potential difficulties with the $(O/C_7)_s$ design leads to design 5. This design has a lay-up of $(\pm 45/C_6)_s$, this laminate being much easier to fabricate than the $(O/C_7)_s$ and the ± 45 layers providing the needed resistance to any unwanted shear. The straightline fiber layers provide a significant resistance to unwanted shear loading and they do contribute some to axial strength. In this case the iteration scheme is used to determine the fiber orientation in the curvilinear layers such that at each point in those layers the fibers are aligned with the principal stress directions in those layers. The maximum strain failure criterion indicates that this design reacts 84% more load than the baseline quasi-isotropic case and it fails due to fiber failure at the net section hole edge. The Tsai-Wu criterion also indicates a substantial strength advantage for this plate. It should be pointed out that at identical x-y locations in the $(C_8)_s$, the $(O/C_7)_s$, and the $(\pm 45/C_6)_s$ plates the fiber directions of the curvilinear layers are not identical. Indeed, at similar points in the plates these three designs constitute laminates with different elastic properties. There is no reason that the principal stress directions should be the same in each case. As might be expected, however, at a particular x-y location the direction of the curvilinear fibers is similar for the three designs.

Though both the maximum strain and the Tsai-Wu criteria predict the $(\pm 45/C_6)_s$ to react less load than orthogonal - curvilinear design 4, it is not a completely fair comparison. Design 4 has 14 curvilinear layers reacting the load whereas design 5 has only 12. To maintain a 16 layer laminate (and thus compare laminates of identical weight), four of the 16 curvilinear layers in the $(C_8)_s$ were used for the off-axis ± 45 layers, leaving 12 for the curvilinear format. To convert the $(C_8)_s$ to the $(O/C_7)_s$ lami-

nate, only two of the 16 layers were used, leaving 14 for the curvilinear format. To affect a fairer comparison, design 6 is considered. This design, like design 5, has 12 loading-bearing curvilinear layers. For design 5 the iteration process results in a laminate that carries twice as much load as the baseline quasi-isotropic laminate, and, according to the maximum strain criterion, about 25% more load than the $(\pm 45/C_6)_s$ design. The criterion predicts failure is due to fiber failure in the curvilinear layers at the net-section hole edge. As can be seen, the Tsai-Wu criterion predicts that designs 5 and 6 react practically the same load, a deviation from the maximum strain criterion prediction.

The design 5 $(\pm 45/C_6)_s$ laminate has a counterpart in the straightline format, namely a $(\pm 45/0_6)_s$ laminate, design 7. Both have four off-axis layers at $\pm 45°$ to provide resistance to shear and both have 12 load-bearing layers. The major difference is that the load-bearing layers are curvilinear in design 5 whereas they are straight in design 7. Application of the maximum strain criterion to the straightline format indicates that it does not react as much load as its curvilinear counterpart, specifically a 1.43 load capacity as compared to a 1.84 load capacity. Failure of the straightline design is predicted to be fiber failure near the net-section hole edge, the same mode as the curvilinear design. Interestingly enough, the Tsai-Wu criterion predicts considerably less load capacity for the straightline design than does the maximum strain criterion. This is felt to be due to the Tsai-Wu criterion predicting an interaction between shear and fiber failures. This has not been thoroughly checked. What is puzzling is the following: Near the net-section hole edge both designs look identical. In the curvilinear design the curvilinear fibers pass by the net section perpendicular to a line from the hole edge to the plate edge. In the straightline design the 0° fibers also pass by the net section perpendicular to that same line. In both designs the $\pm 45°$ layers have identical orientation at the net section hole edge. Locally, then, near the net-section hole edge, where failure is predicted to occur in both cases, the two laminates look identical. Yet their load capacities are different. It seems that the

manner in which the stress trajectories transmit the load past the hole is important. The curvilinear fibers move the load through the net section differently than the straightline fibers. How they are different is still being investigated.

Two more designs which further illustrate this point, and provide other comparisons, are designs 8 and 9. Design 8, with stacking arrangement ($\pm 45/0_2)_{2s}$, is a common laminate which exhibits orthotropic material properties. There are more $\pm 45°$ layers in this design than in design 7, and therefore less load-bearing $0°$ layers. The maximum strain criterion predicts that this design will carry about 30% more load than the baseline quasi-isotropic laminate. This is less than design 7, which carries 43% more than the baseline. The maximum strain criterion predicts that failure is due to fiber failure near the net-section hole edge. The Tsai-Wu criterion predicts that design 8 will carry more than design 7, despite a lesser number of load-bearing fibers. The curvilinear counterpart to design 8 is design 9, a ($\pm 45/C_2)_{2s}$ laminate. The maximum strain criterion predicts that design 9 will carry 47% more than the baseline laminate and about 20% more than design 8, its straightline format counterpart. The maximum strain criterion predicts design 9 will fail by fiber failure at the net-section hole edge. The Tsai-Wu criterion predicts a similar load level for the ($\pm 45/C_2)_{2s}$ laminate and predicts similar gains for this laminate relative to its straightline format counterpart. As with designs 6 and 7, near the net section designs 8 and 9 look identical. Yet the curvilinear format transmits the load around the hole differently than the the straightline format and results in a higher load capacity.

Table 2 presents a comparison among the same 9 designs for the case of a square planform, L/W = 1, and a small hole, D/W = 1/6. In reality the results in table 2 are for a laminate 10 x 10 in. square with a hole 1.67 in. in diameter. The results have been normalized by the quasi-isotropic design for this same geometry, not the geometry of Table 1. The maximum strain failure load for the square quasi-isotropic design is 26.4 Kips/in. The conclusions regarding the failure of the various designs

and the comparisons among designs, particularly the comparisons between the straightline formats and their curvilinear counterparts, are the same as for Table 1. However, for a given design - e.g., a $(\pm 45/C_6)_2$ - the normalized failure load for a square planform is slightly less than the normalized failure load for the rectangular planform. For example, the normalized failure load for design 5 for the rectangular planform is 1.84 while for the square planform it is 1.79. For design 4 the rectangular and square planforms have failure loads of 2.24 and 2.21, respectively. This indicates that the proximity of the hole to the ends of the plate, where the tensile load is applied, is important in determining load capacity. This is indeed the case when one compares the 27.6 Kips/in. failure load of the rectangular planform quasi-isotropic plate with the 26.4 Kips/in. failure load of the square planform isotropic plate.

Tables 3 and 4 indicate that the same improvements in performance using the curvilinear format are realized for the rectangular and square planforms with a larger hole, specifically a value of D/W = 1/3. The results of table 3, which are for values of L/W = 2 and D/W = 1/3, were computed using a plate 20 in. long and 10 in. wide with a hole diameter of 3.333 in. The results of table 4, which are for values of L/W = 1 and D/W = 1/3, were computed using a plate 10 in. square with a hole 3.333 in. in diameter. The results of table 4, for the maximum strain criterion, were the results reported in ref. 1. The trend going from the geometry of table 1 to the geometry of table 2 continues with the geometries of tables 3 and 4. Specifically, for a given design - e.g., design 5 - the normalized failure load for the rectangular plate with the large hole, table 3, is less than the normalized failure load for the square plate with the small hole, table 2 - i.e., 1.71 vs. 1.79. In a similar manner, the normalized load for a given design square plate with the large hole, table 4, is less than the normalized load for that same design but with a small hole, table 3. In going from the geometry of table 1 to the geometry of table 4, a given design carries a decreasing load. However, the results of Tables 1 - 4 indicate unequivocally that the curvilinear philosophy results in tensile load capacities that are greater than conventional

straightline design philosophies. Both failure criteria show this to be the case, and generally both criteria predict the same level of improvement. For completeness, it should be mentioned that the actual failure loads for the baseline cases of tables 3 and 4 were 24.1 Kips/in. and 20.4 Kips/in., respectively.

To further illustrate that the geometry of the plate does influence the load capacity, table 5 presents the data of tables 1-4 in a slightly different format. In table 5 the failure loads predicted by the two criteria are normalized by the failure load of the rectangular quasi-isotropic plate with the small hole. This is design 1 in table 1. From this table it is clear that as the geometry of the plate changes, in going from left to right, the strength of a given design decreases. Taking extremes as an example, design 5 for the rectangular plate with the small hole (L/W = 2 and D/W = 1/6) fails, according to the maximum strain criterion, at a normalized load of 1.84. The same design for a square plate with a large hole (L/W = 1 and D/W = 1/3) fails at 1.18. The latter plate fails at a load 36% lower than the former plate. This is strictly a geometric effect.

Resistance to Buckling - Preliminary Results

The pressing question is the effect of the curvilinear format on the compression re-sponse of the plates. Since these plates are quite thin relative to their planform area, out-of-plane buckling is expected to be the primary compressive response. And since flat plates have a measure of post-buckling strength, computing the buckling load of the plate based on buckling displacements relative to the flat prebuckling state is a credible measure of buckling resistance. The primary question is: Is the curvilinear format which enhances inplane tensile capacity also an advantage as far as buckling capacity is concerned? An a priori assessment is as follows: If the buckling load increases due to the curvilinear format, then it is clear that the curvilinear format is a better plate design. Both tensile and compressive perform-ance have been improved. On the otherhand, if the buckling load decreases, then it is not clear whether the curvilinear format is better. It depends on the application of the plate. However, since most structural components must react both tension and compression, a decrease in the buckling load could be detrimental to the overall gain in performance. Finally, if the buckling load does not change significantly, then the curvilinear design is an improvement. The tensile capacity has been improved at little or no sacrifice to the compressive capacity.

The buckling loads for several designs of the square planform plate with the large hole (L/W = 1 and D/W = 1/3, table 4) were computed with the commercially avail-able finite-element code EAL [2]. Table 6 shows the limited results. The table shows the normalized buckling loads of the first 6 designs. The actual loads are normalized

by the buckling load of the baseline quasi-isotropic design. An examination of the table indicates that none of the designs, curvilinear or straightline format, reacts as high a buckling load as the baseline quasi-isotropic design. Design 4, while showing superior tensile resistance (roughly a factor of 2 over the baseline design), buckles at 41% lower load than the baseline design. Interestingly enough, however, design 5 shows only an 11% decrease in the buckling load. This design was the preferred design in the tensile study because it can react unwanted shear and still show an improvement over the baseline case. Because of the large increase in tensile capacity and minimal decrease in buckling resistance, it is felt that this design is indeed a good design.

Further buckling studies are underway to assess the other designs discussed in tables 1-5. In addition, efforts are being made to understand the mechanics of the load transfer around the hole to explain the above-mentioned difference in the tensile load capacity between design 5 and design 7, and 8 and 9, seemingly similar designs when the local region around the net section hole edge is isolated.

References

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TABLE 1 - NORMALIZED TENSILE FAILURE LOAD[1]
L/W = 2; D/W = 1/6

Design No.	Stacking Arrangement	Failure Load Max. Strain (Tsai-Wu)	Failure Mode (Based on Max. Strain Criterion)
1	$(\pm 45/0/90)_{2s}$	1.00 (0.99)	Fiber failure in 0° layers at net-section hole edge
2	$(0_8)_s$	0.83 (0.67)	Shear failure in matrix at hole near net-section
3	$(C_8)_s$	1.26 (1.10)	Tension perpendicular to fiber direction at net-section away from hole edge
4	$(O/C_7)_s$	2.29 (1.94)	Fiber failure in curvilinear layers at net-section hole edge (Orthogonal layers eliminate problem of tensile failure in matrix; This laminate achieves the highest fiber-direction tensile stress in the curvilinear layers)
5	$(\pm 45/C_6)_s$	1.84 (1.91)	Fiber failure in curvilinear layers at net-section hole edge (Compared to $(\pm 45/C_2)_{2s}$ the greater number of curvilinear layers increases the longitudinal tensile strength and the shear strength of the curvilinear layers despite the fewer number of +45° and -45° plies; The problem of shear failure in the matrix is eliminated)
6	$(O_2/C_6)_s$	2.09 (1.93)	Fiber failure in curvilinear layers at net-section hole edge
7	$(\pm 45/0_6)_s$	1.43 (1.11)	Fiber failure in 0° layers at net-section hole edge (Shear failure at hole near net-section, Tsai-Wu indicates interaction between shear failure and fiber failure)
8	$(\pm 45/0_2)_{2s}$	1.29 (1.32)	Fiber failure in 0° layers at net-section hole edge (Tsai-Wu indicates interaction between shear failure and fiber failure)
9	$(\pm 45/C_2)_{2s}$	1.47 (1.52)	Fiber failure in curvilinear layers at net-section hole edge

[1] normalized by design 1, maximum strain criterion.

TABLE 2 - NORMALIZED TENSILE FAILURE LOAD[2]
L/W = 1; D/W = 1/6

Design No.	Stacking Arrangement	Failure Load Max. Strain (Tsai-Wu)	Failure Mode (Based on Max. Strain Criterion)
1	$(\pm 45/0/90)_{2s}$	1.00 (0.99)	Fiber failure in 0° layers at net-section hole edge
2	$(0_8)_s$	0.76 (0.62)	Shear failure in matrix at hole near net-section
3	$(C_8)_s$	1.18 (1.04)	Tension perpendicular to fiber direction at net-section away from hole edge
4	$(O/C_7)_s$	2.21 (1.87)	Fiber failure in curvilinear layers at net-section hole edge (Orthogonal layers eliminate problem of tensile failure in matrix; This laminate achieves the highest fiber-direction tensile stress in the curvilinear layers)
5	$(\pm 45/C_6)_s$	1.79 (1.85)	Fiber failure in curvilinear layers at net-section hole edge (Compared to $(\pm 45/C_2)_{2s}$ the greater number of curvilinear layers increases the longitudinal tensile strength and the shear strength of the curvilinear layers despite the fewer number of +45° and -45° plies; The problem of shear failure in the matrix is eliminated)
6	$(O_2/C_6)_s$	2.03 (1.87)	Fiber failure in curvilinear layers at net-section hole edge
7	$(\pm 45/0_6)_s$	1.38 (1.07)	Fiber failure in 0° layers at net-section hole edge (Shear failure at hole near net-section, Tsai-Wu indicates interaction between shear failure and fiber failure)
8	$(\pm 45/0_2)_{2s}$	1.26 (1.29)	Fiber failure in 0° layers at net-section hole edge (Tsai-Wu indicates interaction between shear failure and fiber failure)
9	$(\pm 45/C_2)_{2s}$	1.44 (1.49)	Fiber failure in curvilinear layers at net-section hole edge

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[2] normalized by design 1, maximum strain criterion.

TABLE 3 - NORMALIZED TENSILE FAILURE LOAD[3]
L/W = 2; D/W = 1/3

Design No.	Stacking Arrangement	Failure Load Max. Strain (Tsai-Wu)	Failure Mode (Based on Max. Strain Criterion)
1	$(\pm 45/0/90)_{2s}$	1.00 (0.99)	Fiber failure in 0° layers at net-section hole edge
2	$(0_8)_s$	0.68 (0.59)	Shear failure in matrix at hole near net-section
3	$(C_8)_s$	1.28 (1.11)	Tension perpendicular to fiber direction at net-section away from hole edge
4	$(O/C_7)_s$	2.04 (1.92)	Fiber failure in curvilinear layers at net-section hole edge (Orthogonal layers eliminate problem of tensile failure in matrix; This laminate achieves the highest fiber-direction tensile stress in the curvilinear layers)
5	$(\pm 45/C_6)_s$	1.71 (1.77)	Fiber failure in curvilinear layers at net-section hole edge (Compared to $(\pm 45/C_2)_{2s}$ the greater number of curvilinear layers increases the longitudinal tensile strength and the shear strength of the curvilinear layers despite the fewer number of +45° and -45° plies; The problem of shear failure in the matrix is eliminated)
6	$(O_2/C_6)_s$	1.90 (1.76)	Fiber failure in curvilinear layers at net-section hole edge
7	$(\pm 45/0_6)_s$	1.38 (1.06)	Fiber failure in 0° layers at net-section hole edge (Shear failure at hole near net-section, Tsai-Wu indicates interaction between shear failure and fiber failure)
8	$(\pm 45/0_2)_{2s}$	1.28 (1.31)	Fiber failure in 0° layers at net-section hole edge (Tsai-Wu indicates interaction between shear failure and fiber failure)
9	$(\pm 45/C_2)_{2s}$	1.42 (1.47)	Fiber failure in curvilinear layers at net-section hole edge

[3] normalized by design 1, maximum strain criterion.

TABLE 4 - NORMALIZED TENSILE FAILURE LOAD[4]
L/W = 1; D/W = 1/3

Design No.	Stacking Arrangement	Failure Load Max. Strain (Tsai-Wu)	Failure Mode (Based on Max. Strain Criterion)
1	$(\pm 45/0/90)_{2s}$	1.00 (0.99)	Fiber failure in 0° layers at net-section hole edge
2	$(0_8)_s$	0.59 (0.51)	Shear failure in matrix at hole near net-section
3	$(C_8)_s$	1.01 (0.93)	Tension perpendicular to fiber direction at net-section away from hole edge
4	$(O/C_7)_s$	1.89 (1.78)	Fiber failure in curvilinear layers at net-section hole edge (Orthogonal layers eliminate problem of tensile failure in matrix; This laminate achieves the highest fiber-direction tensile stress in the curvilinear layers)
5	$(\pm 45/C_6)_s$	1.60 (1.66)	Fiber failure in curvilinear layers at net-section hole edge (Compared to $(\pm 45/C_2)_{2s}$ the greater number of curvilinear layers increases the longitudinal tensile strength and the shear strength of the curvilinear layers despite the fewer number of +45° and -45° plies; The problem of shear failure in the matrix is eliminated)
6	$(O_2/C_6)_s$	1.79 (1.66)	Fiber failure in curvilinear layers at net-section hole edge
7	$(\pm 45/0_6)_s$	1.27 (0.94)	Fiber failure in 0° layers at net-section hole edge (Shear failure at hole near net-section, Tsai-Wu indicates interaction between shear failure and fiber failure)
8	$(\pm 45/0_2)_{2s}$	1.20 (1.22)	Fiber failure in 0° layers at net-section hole edge (Tsai-Wu indicates interaction between shear failure and fiber failure)
9	$(\pm 45/C_2)_{2s}$	1.33 (1.38)	Fiber failure in curvilinear layers at net-section hole edge

[4] normalized by design 1, maximum strain criterion.

TABLE 5 - NORMALIZED TENSILE FAILURE LOAD[1]
COMPARISON AMONG GEOMETRIES

Design No.	Stacking Arrangement	L/W=2;D/W=1/6 Failure Load Max.Str.(T-Wu)		L/W=1;D/W=1/6 Failure Load Max.Str.(T-Wu)		L/W=2;D/W=1/3 Failure Load Max.Str.(T-Wu)		L/W=1;D/W=1/3 Failure Load Max.Str.(T-Wu)	
1	(±45/0/90)$_{2s}$	1.00	(0.99)	0.96	(0.95)	0.88	(0.87)	0.74	(0.73)
2	(0$_8$)$_s$	0.83	(0.67)	0.72	(0.59)	0.59	(0.52)	0.43	(0.38)
3	(C$_8$)$_s$	1.26	(1.10)	1.13	(1.00)	1.13	(0.98)	0.75	(0.69)
4	(O/C$_7$)$_s$	2.29	(1.94)	2.11	(1.79)	1.79	(1.68)	1.40	(1.32)
5	(±45/C$_6$)$_s$	1.84	(1.91)	1.71	(1.77)	1.50	(1.55)	1.18	(1.23)
6	(O$_2$/C$_6$)$_s$	2.09	(1.93)	1.94	(1.79)	1.67	(1.54)	1.32	(1.23)
7	(±45/0$_6$)$_s$	1.43	(1.11)	1.32	(1.03)	1.21	(0.93)	0.94	(0.70)
8	(±45/0$_2$)$_{2s}$	1.29	(1.32)	1.21	(1.24)	1.12	(1.15)	0.89	(0.90)
9	(±45/C$_2$)$_{2s}$	1.47	(1.52)	1.38	(1.43)	1.25	(1.29)	0.99	(1.02)

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[1] normalized by design 1, L/W = 2 D/W = 1/6, maximum strain criterion.

TABLE 6 - BUCKLING LOADS[1]
L/W = 1; D/W = 1/3

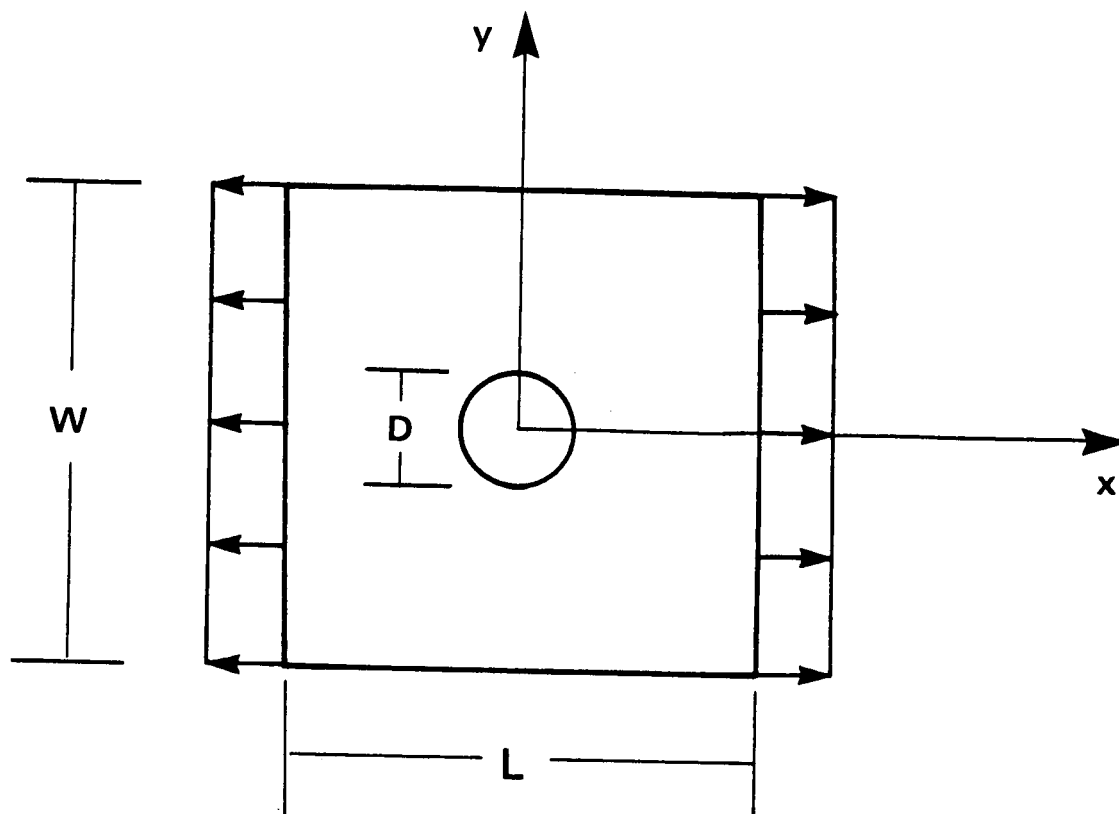| Design No.	Stacking Arrangement	Buckling Load
1	$(\pm 45/0/90)_{2s}$	1.00
2	$(0_8)_s$	0.53
3	$(C_8)_s$	0.57
4	$(O/C_7)_s$	0.59
5	$(\pm 45/C_6)_s$	0.89
6	$(O_2/C_6)_s$	0.50

[1] normalized by design 1.

Fig. 1 Plate Geometry and Nomenclature.
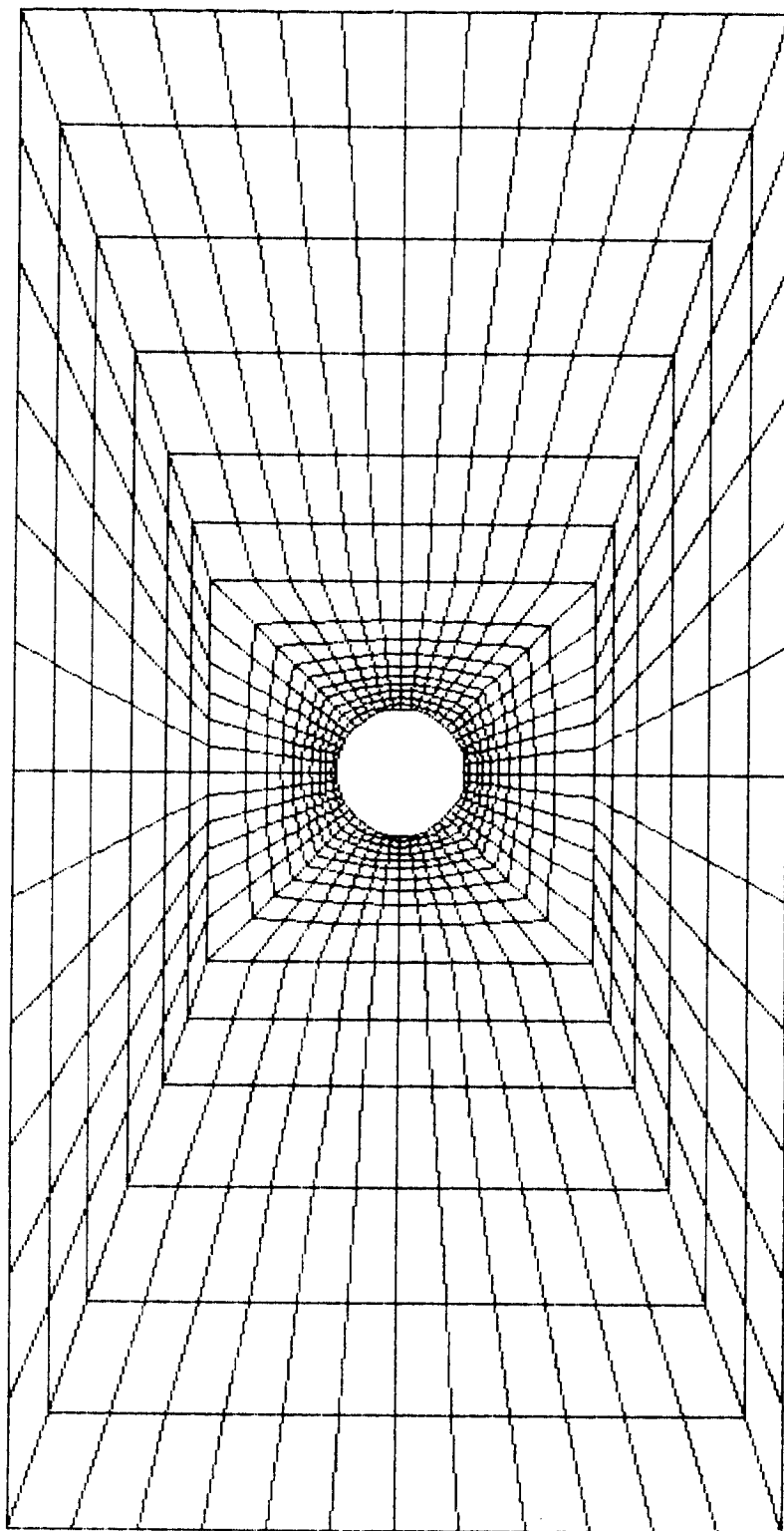
Fig. 2 Finite-Element Mesh for Case L/W = 2, D/W = 1/6.
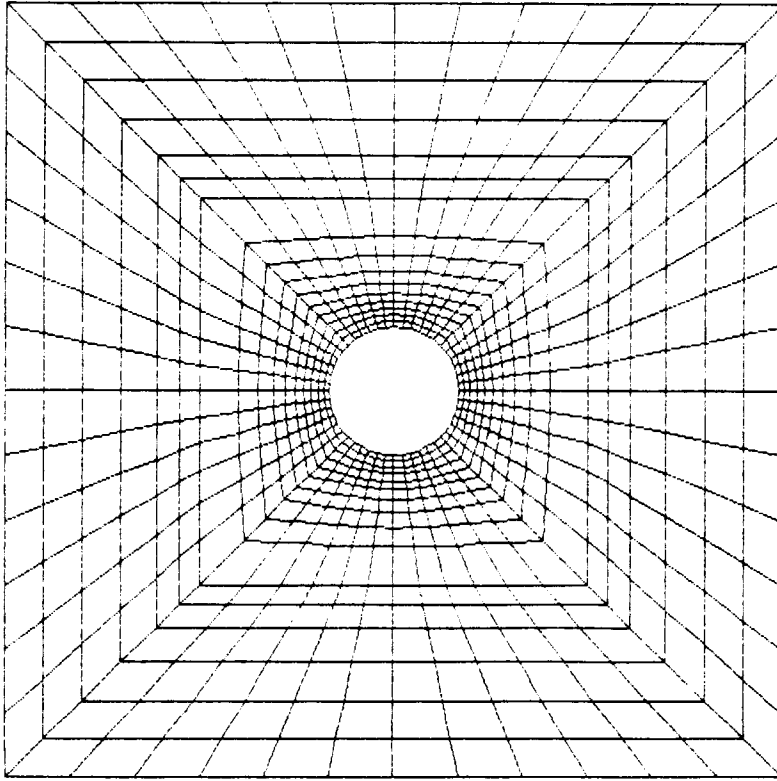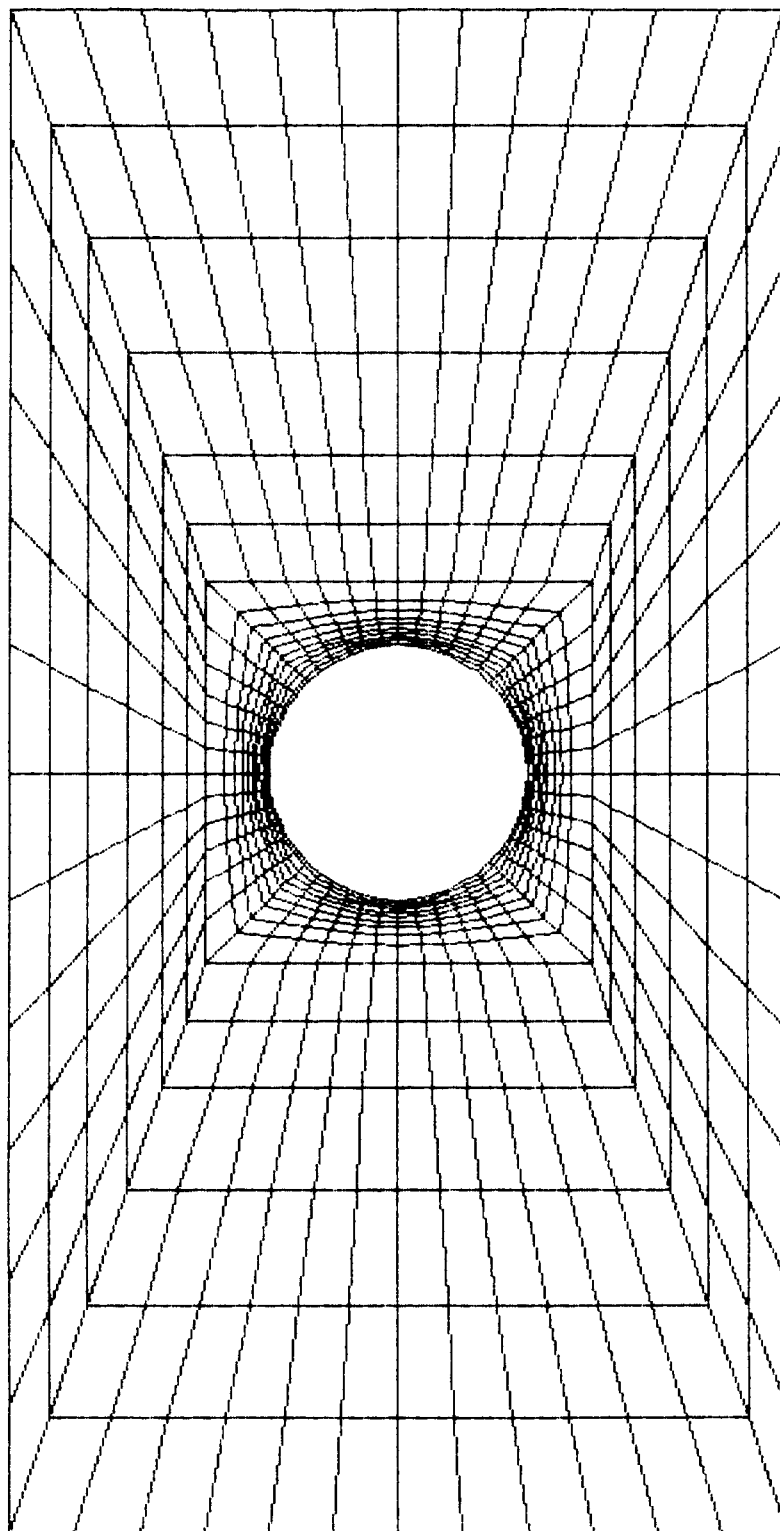
Fig. 3 Finite-Element Mesh for Case L/W = 1, D/W = 1/6.
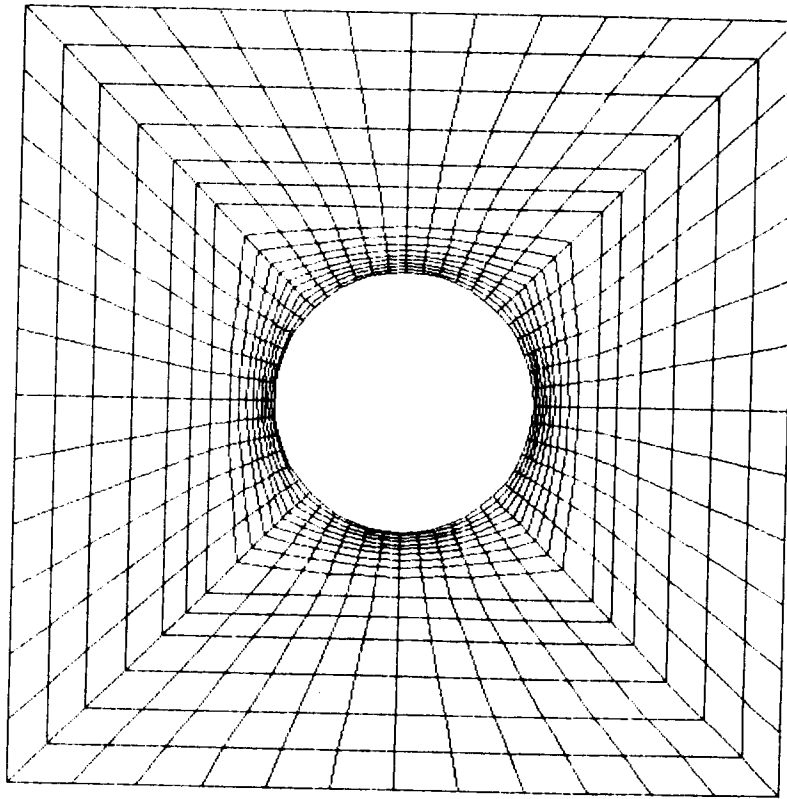
Fig. 4 Finite-Element Mesh for Case L/W = 2, D/W = 1/3.

Fig. 5 Finite-Element Mesh for Case L/W = 1, D/W = 1/3.

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15. Supplementary Notes

16. Abstracts

Further studies to determine the potential for using a curvilinear fiber format in the design of composite laminates are reported on. The curvilinear format is in contrast to the current practice of having the fibers aligned parallel to each other and in a straight line. The problem of a plate with a central circular hole is used as a candidate problem for this study. The study concludes that for inplane tensile loading the curvilinear format is superior. The limited results to date on compression buckling loads indicate that the curvilinear designs are poorer in resistant buckling. However, for the curvilinear design of interest, the reduction in buckling load is minimal and so overall there is a gain in considering the curvilinear design.

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17b. Identifiers/Open-Ended Terms

curvilinear fiber format, structural efficiency, tensile strength, buckling resistance, centrally located hole

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